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Five Property Types' Real Estate Cycles as Markov Chains

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Metro market real estate cycles for office, industrial, retail, apartment, and hotel properties may be specified as first order Markov chains, which allow analysts to use a well-developed application, "staying time". Anticipations for time spent at each cycle point are consistent with the perception of analysts that these cycle changes speed up, slow down, and pause over time. We find that these five different property types in U.S. markets appear to have different first order Markov chain specifications, with different staying time characteristics. Each of the five property types have their longest mean staying time at the troughs of recessions. Moreover, industrial and office markets have much longer mean staying times in very poor trough conditions. Most of the shortest mean staying times are in hyper supply and recession phases, with the range across property types being narrow in these cycle points. Analysts and investors should be able to use this research to better estimate future occupancy and rent estimates in their discounted cash flow (DCF) models.

Keywords

Real Estate Cycle, Markov Chain, Commercial Real Estate, Staying Times

1. Summary and Main Results

Real estate cycle conditions may be modeled as first order Markov chains across markets for investments in office, industrial, retail, apartment, and hotel properties. This means that probabilities across cycle points in the future may be generated with prior knowledge of only initial cycle position and prior history of the probabilities for quarter-to-quarter transitions. This research analyzes those transitions based on historic cycle movements. The null hypothesis that the processes are zero order models may be rejected with standard test statistics, and there does not appear to be value in adding the complexity of a second order model.

We find that the five different property types have different first order models, with tests that show that pooling the property type samples lowers the explanatory power of the model— thus differences in the data generating processes may offset gains from combining samples from pairs of different property types. However, our tests did justify the pooling of large and small market subsamples in each of the five property markets. Finally, the cycle stage of one property type in a city does not appear to be a covariate for other property types in that city to the degree that extra model complexity gives compensating gains in prediction success.

A standard Markov chain calculation allows us to generate the probability distribution for the number of quarters that a city market might stay at its initial cycle point. "Staying time" changes across property types and initial cycle conditions. The distributions are known to be geometric distributions, which give easily calculated means and standard deviations, reported here for each property type. Mean staying time shortens and lengthens over the cycle, consistent with the perception by many real estate observers that changes in markets speed up and slow down over the real estate cycle.

Each of the five property types have their longest mean staying time at the troughs of recessions. Moreover, industrial and office markets have much longer mean staying times in very poor trough conditions. These property types are less attractive in those cycle points than other property types that have mean staying times that are half or one third of office and industrial. On the other hand, the mean staying times of office and industrial are the most attractive among the set of five for the most profitable cycle point that represents the highest occupancies and rent conditions, Cycle Point 11. Most of the shortest mean staying times are in hyper supply and recession phases, with the range across property types being narrow in these cycle points.

A review of the real estate cycle literature appeared recently (Evans and Mueller, 2013). Mueller began to produce his market cycle occupancy analysis in 1992 and published his theory in 1995. The occupancy cycle model in Mueller (1995), represented by a stylized sine wave curve which uses sixteen points on the cycle curve, has remained unchanged for the past 22 years.

The format used in Figure 1 allows a concise presentation of the cycle points of more than fifty markets for a given quarter, thus allowing the reader to distinguish between larger and smaller markets. The information of each market allows the reader to see that it has stayed at the same point that existed in the prior quarter, moved right, or moved left in the cycle representation. This paper depends on that model to generate a cycle representation that is in the format of a Markov chain model of probabilistic change between cycle points quarter-to-quarter. The Markov chain charts for the five property types appear as Figures 2 through 6. These figures omit some transition probabilities that do not round to .03, but these are reported to four decimal places in Appendix Tables 1 through 5.

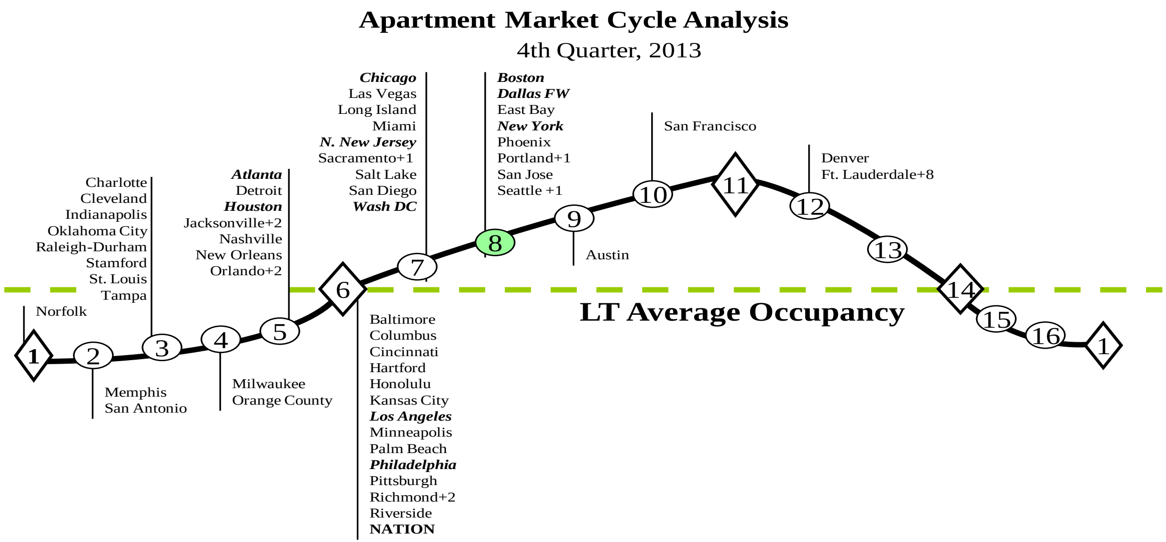
The plan of this paper begins with a review of Markov chain models as applied to real estate cycles. In the second section, the real estate cycle data and the sources used here are described; in this same section, an explanation is given of the essential tally transition matrices for each of the five property types, provided in Appendix Tables 1 through 5. In the third section, tests on the samples used here are reported to establish the specification of the models to be applied. The final sections demonstrate the major applications of the model, which show notable differences across property types and stages of the cycles with respect to how long the market conditions pause before they show qualitative changes.

2. Markov Chain Definitions and Descriptions

We list the sixteen alternative real estate cycle point states in vector notation as $(s_1 \; s_2 \; \ldots \; s_{16})$. Some of the most useful predictions and key inputs to the analysis come with another kind of vector, one that gives the distribution of probabilities across alternative states. This is a probability vector, p^n, for a period n steps ahead, $p^n = (p_1^n \; p_2^n \; p_3^n \ldots p_{16}^n)$. In a probability vector, the sum of the elements equals one, and each element is non-negative. For example, through the use of quarterly analysis, the forecast might give the probability of a real estate market being in alternative cycle points four quarters ahead. In vector p^4, the element p_i^4 gives the probability that the process will be in s_i after four periods of possible change.

Another probability vector is an analytical input, one that describes a current period--zero steps ahead, $p^0 = (p_1^0 \; p_2^0 \; p_3^0 \ldots p_{16}^0)$. Initial conditions are described by p^0 with considerable flexibility, but all the examples considered in this paper use a case in which the initial state is known with certainty.

Figure 1 Apartment Market Cycle Analysis from Real Estate Cycle Monitor



Source: Mueller, 2014

Figure 2 A Markov Chain Representation of the Real Estate Cycle Quarter-to-Quarter Changes: Apartment Markets

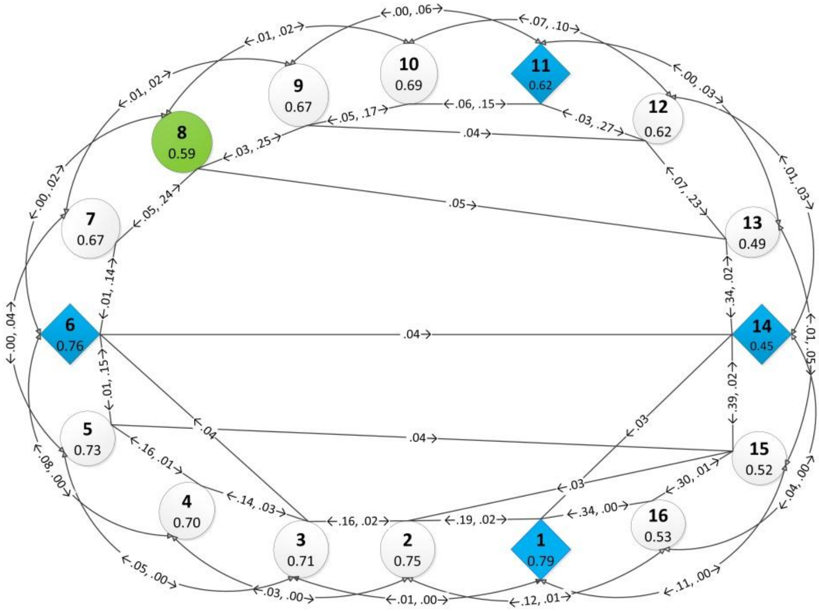


Figure 3 A Markov Chain Representation of the Real Estate Cycle Quarter-to-Quarter Changes: Hotel Markets

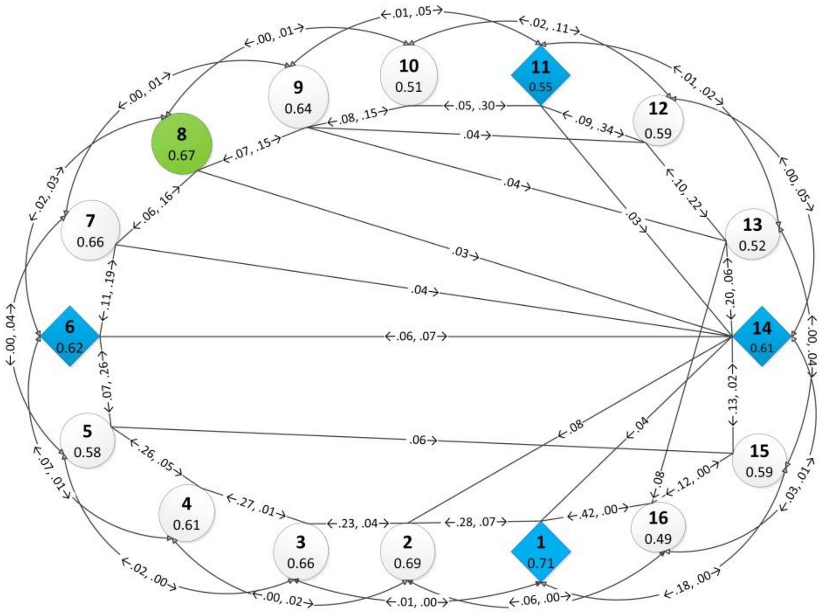


Figure 4 A Markov Chain Representation of the Real Estate Cycle Quarter-to-Quarter Changes: Industrial Markets

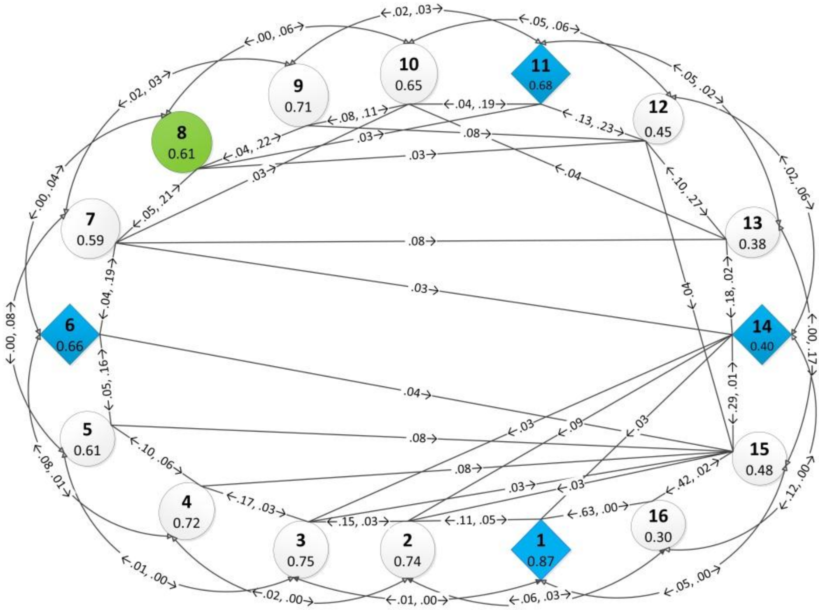


Figure 5 A Markov Chain Representation of the Real Estate Cycle Quarter-to-Quarter Changes: Office Markets

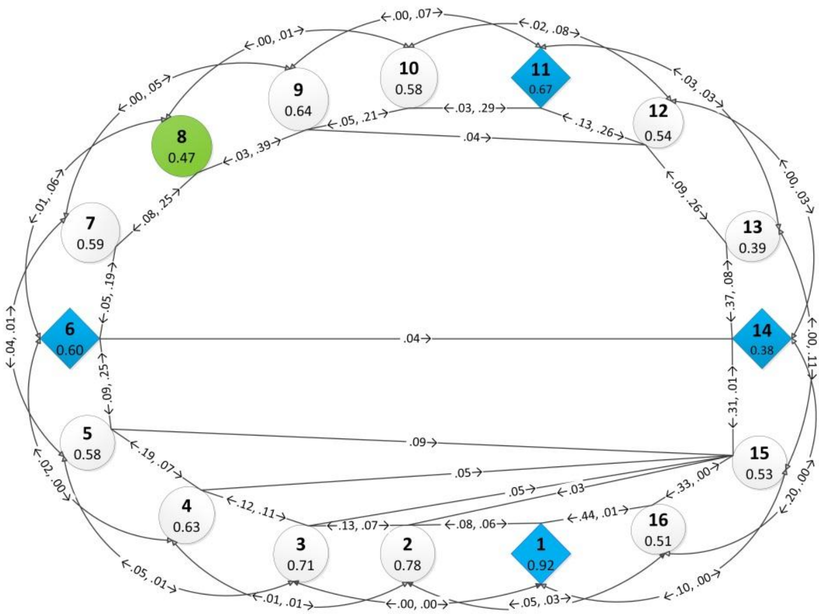
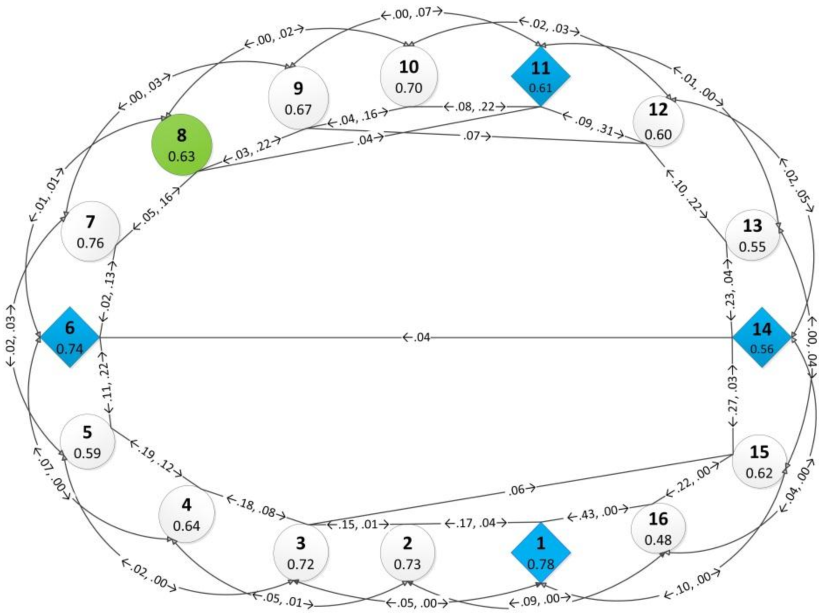


Figure 6 A Markov Chain Representation of the Real Estate Cycle Quarter-to-Quarter Changes: Retail Markets



A second set of input data in a Markov chain analysis gives transition probabilities. We define $p_{i,j}$ as the probability that a market that is at cycle point s_i in any given quarter is then in s_j in the next quarter. These probabilities can be fully listed in a transition matrix, P, a square matrix with non-negative elements such that the sum of each row is one.

$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,16} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,16} \\ . & . & . & . \\ p_{16,1} & p_{16,2} & \cdots & p_{16,16} \end{bmatrix} \tag{1}$$

For a first order Markov chain, the set of sixteen cycle point probabilities k periods ahead, p^k, are calculated from the probabilities that alternative states exist in period k-1 and the probabilities of transition among states. For a one step ahead forecast

$$\begin{aligned} p_1^k &= p_1^{k-1}p_{1,1} + p_2^{k-1}p_{2,1} + p_3^{k-1}p_{3,1} + \cdots + p_{16}^{k-1}p_{16,1} \\ p_2^k &= p_1^{k-1}p_{1,2} + p_2^{k-1}p_{2,2} + p_3^{k-1}p_{3,2} + \cdots + p_{16}^{k-1}p_{16,2} \\ p_3^k &= p_1^{k-1}p_{1,3} + p_2^{k-1}p_{2,3} + p_3^{k-1}p_{3,3} + \cdots + p_{16}^{k-1}p_{16,3} \\ & \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ p_{16}^k &= p_1^{k-1}p_{1,16} + p_2^{k-1}p_{2,16} + p_3^{k-1}p_{3,16} + \cdots + p_{16}^{k-1}p_{16,16} \end{aligned} \tag{2}$$

the matrix expression is much more compact: $p^k = p^{k-1} P$.

The elements of the transition matrix, P, may be established by following several approaches that are each conceptually valid, according to the practitioners of Markov chain analysis. It is perfectly valid to specify them subjectively, or with theoretical arguments, or with common sense and judgment. Empirical and theoretical probability models can sometimes give the elements.

With the data available for this study, inference and empirically estimating the transition matrix may be directly approached by collecting data on the history of quarter-to-quarter changes of state (cycle point location) observed over many periods and multiple cities. A tally matrix can describe the frequency—the count—observed that the sample set of markets made specific, one-quarter transitions over the sample period. The count of transitions from state i to s_j is $f_{i,j}$, while the marginal count $f_{i,.}$ is the sum of that row's frequencies, the total count of observed transitions that began in s_i. The marginal count $f_{.,j}$ is the sum of the frequencies of that column, the total count of observations that ended in s_j. The total sample size of observed transitions is $f_{.,.}$.

$$
\begin{array}{cccccc}
 & s_1 & s_2 & \cdots & s_{16} & \\
s_1 & f_{1,1} & f_{1,2} & \cdots & f_{1,16} & f_{1,.} \\
s_2 & f_{2,1} & f_{2,2} & \cdots & f_{2,16} & f_{2,.} \\
 & & \cdot & \cdot & \cdot & \\
s_{16} & f_{16,1} & f_{16,1} & \cdots & f_{16,16} & f_{16,.} \\
 & f_{.,1} & f_{.,2} & \cdots & f_{.,16} & f_{.,.}
\end{array}
$$

Maximum likelihood estimators for the transition probabilities may be calculated from the tally matrix as the relative frequency across the $f_{i,.}$ instances that were initially in state i that saw a transition from s_i to s_j:

$$\hat{p}_{i,j} = \frac{f_{i,j}}{f_{i,.}}. \quad (3)$$

Given that the transition probabilities do not change over time, Anderson and Goodman (1957) show that the estimators are consistent, which means that their bias decreases as sample size increases.

3. Data

3.1 Data for Tally and Transition Matrices

Cycle charts such as those seen in Figure 1 follow the model developed by Mueller (1995) and currently published by Dividend Capital Research. The *Real Estate Market Cycle Monitor* reports on current market conditions in 54 markets; we are able to use long data histories on individual markets in up to 53 of those markets, which vary by property type. The full sample used here covers the periods between the fourth quarter of 1996 and the fourth quarter of 2012. Subsamples were also analyzed for the smaller markets of each property type versus the largest markets. The largest markets were determined as those that make up 50% of all the square footage in the 54 market sample. It takes between 11 and 14 markets to make up the 50%, depending on the property type. Those markets are indicated with bold italic print fonts in the charts. The five property types are office, industrial, apartment, retail and hotel.

Cycle Point 1 in Mueller's model represents the trough of recession—lowest occupancy rates, and low and declining rental rates. Cycle Points 2—5 represent the recovery phases of the real estate cycle—improving occupancy rates (that are still below long term average for each particular city) and rental rates that are either declining or growing more slowly than inflation. Cycle Point 6 marks the long term occupancy average with rents that are growing at the same rate as inflation. This also marks the beginning point of the expansion phase of the real estate cycle with above average occupancy rates and rents that are growing faster than inflation. A key point of interest is Cycle Point 8, the midpoint of the expansion phase where cost feasible new construction rents are reached. Cycle Point 11 has a key interpretation as the peak of the cycle with the highest occupancy level. It is also known as economic equilibrium as demand and supply are growing at the same rate. It is the precursor to the hyper-supply cycle phase, where while occupancy is high, new supply is growing faster than demand, thus decreasing occupancy and causing rental rate growth to slow. The recession phase begins after Cycle Point 14 as occupancy crosses to below its long term average, and construction completions begin to more seriously worsen supply problems. In the recession phase, rental growth rates are again below inflation at Cycle Point 15 or negative at Cycle Point 16 then back to Cycle Point 1, the bottom of the cycle.

3.2 Tally Matrices

The real estate market cycle point histories published in past *Real Estate Cycle Monitors* and their precursors provide the raw data to generate the tally matrices here. The frequencies in the tally matrix are the simple count of the number of times in adjacent quarters that any metro market is observed to be transitioning from one cycle point to each possible cycle point. The frequencies may be generated with fairly complex conditional counting spreadsheet functions from a spreadsheet of every city's cycle point history. Given the worry of making spreadsheet errors, the tally matrices reported here were validated by using commercial software (Berchtold 2006). Many of the data functions and model estimates reported here are done with Berchtold's Markov chain software, MARCH v. 3.00, which may be purchased or borrowed on line at http://www.andreberchtold.com. <<Link tested September 12, 2014>> The software does impose some limits that are inconvenient, such as being unable to process data on some city markets that do not have the same, complete data history as other markets.

4. Empirical Tests

4.1 Empirical Tests to Specify the Order of the Markov Chain Model

If the cyclical condition of a real estate property market is generated by a zero order Markov chain process, then the market is randomly determined each quarter, but no extra benefit to a forecaster comes from knowing a priori the market cycle conditions of a quarter. A simple example of a zero order Markov chain would be to repeatedly roll a die with six discrete states possible for each roll. If the die was “fair”, each state would be equally likely, and the transition probabilities could be established with theoretical probability models. If the die was “loaded”, we could keep empirical tallies of the process and, perhaps, win great profit by having empirical knowledge of the probabilities. However, in neither case could we improve these predictions for a future roll if we had extra information--knowing what the prior roll had yielded. If a real estate market was a case of a zero order Markov chain process, then the real estate forecaster would be just as interested in the estimated probabilities as a gambler would be interested in the estimates from watching a loaded die over repeated rolls.

A first order Markov chain is used as the example in a prior section of this paper. With this model, a forecaster may better predict the probabilities of alternative cycle states in one quarter by knowing the transition probabilities among cycle points across two quarter spans and having information on the cycle state that existed in the quarter just before the forecast quarter. A second order Markov chain model is justified if the predictions of conditions one step ahead are improved by knowing what cycle conditions were in the two prior quarters and the transition probabilities that span three quarters.

If the real estate cycle across sixteen points is a zero order Markov chain, then the tally matrix would boil down to have sixteen elements, while there would be 256--that is, (16)(16)-- elements in the tally matrix of a first order model. There are 4,096 elements in a tally matrix of a second order Markov chain--that is, (16)(16)(16). While three dimensional matrices are possible in most spreadsheet software packages, Markov theorists have simplified their representations by showing that a second (or higher) order model may always be alternatively represented by a matrix with, in our case, 256 rows with labels such as “s_h, s_i”, and sixteen columns labeled s_j. The tally elements, $f_{h,i,j}$, are the counts of instances that local markets showed the particular progression, first s_h, then s_i, and then s_j.

The statistical testing is not unlike another large area of statistics, contingency table analysis. Empirical researchers usually worry about whether they will have a large enough samples to have power to distinguish between alternative hypotheses. In contingency table analysis, a rule of thumb that is commonly accepted is that the sample is too small if the expected number of observations is less than five per cell, under the extreme assumption that all cells are equally likely. By using that rule of thumb here, if a real estate cycle is a zero order model with sixteen states, then the sample must be at least 80, (16)(5). If the Markov model is a first order model, then the sample size is too small if it is not 1,280, (16)(16)(5). A sample size of 20,480 observed transitions from s_h to s_i, and then to s_j would be required to meet the rule of thumb for a second order Markov model, (16)(16)(16)(5). The sample sizes, reported in the tally matrices of the five property types, range from 3,276 to 3,465. By using the rule of thumb, we may rely on models of zero and first order, but we should not be highly confident in estimates of second order Markov models. The pooling of all five property types into one sample, if justified, would still fall short of the rule of thumb required sample size to estimate a second order Markov chain model.

Under the null hypothesis that the underlying process is a zero order Markov chain with n possible stages, Anderson and Goodman show that the test statistic, -2 *ln* λ, has an asymptotic χ^2 distribution with $(n-1)^2$ degrees of freedom.

$$-2 ln\,\lambda = 2\sum_{i=j}^{n}\sum_{j=i}^{n} f_{i,j}\, ln\frac{f_{i,j}\,f_{.,.}}{f_{i,.}\,f_{.,j}} = -2ln\prod_{i,j}\left(\frac{\hat{p}_j}{\hat{p}_{i,j}}\right)^{f_{i,j}}$$

The test is essentially a test that $p_{1,j} = p_{2,j} = p_{3,j} = p_{16,j} = p_j$ for all j. Under that null hypothesis, no gain is won by knowing that s_i was the prior state of the process that yielded s_j. The accepting of the null hypothesis would deter our use of many, but not all, of the applications of the Markov chain model in real estate applications. (A gambler can profit from knowing the probabilities of a loaded die.) With 16 cycle points that give us 225 degrees of freedom, the critical value of the χ^2 distribution is 277.3 for a test at the .01 level of the null hypothesis that there is a zero order Markov chain, against the alternative that there is some higher order. See Table 1.1 for the sample test statistic of each property type. In each case, we reject the null hypothesis that the process that generated the sample is a zero order Markov chain.

The testing of the null hypothesis that the Markov chain is of order one against the alternative that it is of a higher order may be done with the test in Anderson and Goodman (page 100) that is based on counting the instances that markets progressed through three quarters, which change from s_h to s_i, and then to s_j, labeled $f_{h,i,j}$. The test is essentially a test that $p_{1,i,j} = p_{2,i,j} = p_{3,i,j} = \ldots = p_{16,i,j} = p_{i,j}$ for all i and j. Under that null hypothesis, no gain is won in forecasting s_j by knowing that s_h was a state of the process two steps prior. The test statistic is asymptotically χ^2 with $n(n-1)^2$ degrees of freedom--3,600 when n = 16. None of the property type samples of the three quarter transition sequences have a sample size that is as large as the number of degrees of freedom in the standard test. None of the property types have a sample size that would meet the rule of thumb for a per-cell expected frequency of at least five.

Table 1 Tests for Model Specification and Ability to Pool Sub-Samples

	Apartment	**Hotel**	**Industrial**	**Office**	**Retail**
1.1: Sample test statistics -2λ to test H_0: Markov chain is of order 0; against H_a: Order is higher. Critical value in a test at the 1% level with 225 d.f. is 277.3; 'CHIINV(.01,15*15)' **Results:** All sample test statistics exceed the critical value for rejecting H.					
Sample -2λ	11,202	10,567	10,742	10,716	11,286
1.2: Sample Bayesian information criterion for estimated models of alternative orders. **Results:** A first order model minimizes the BIC for each property type.					
Order 1	7,353	7,639	6,774	6,063	7,346
Order 2	7,838	8,126	7,239	6,445	7,672
1.3: Pooled Sample Chi Square Tests Statistics H_0: a pair of property types come from the same 1st order Markov chain process; against H_a: the pair come from different 1st order Markov chain process. Critical value in a test at the 95% level with 240 d.f. = 205.1, "CHIINV(.95, 16*15)" **Results:** All sample test statistics exceed the critical value for rejecting H.					
Apartment	--	345.3	269.0	284.9	234.3
Hotel	345.3	--	377.2	387.3	309.7
Industrial	269.0	377.2	--	21.5	303.4
Office	284.9	387.3	21.5	--	29.7
Retail	234.3	309.7	303.4	29.7	--
H_0: size-based sub samples within a property type come from the same 1st order Markov chain process; against H_a: the subsamples come from different 1st order Markov chain process Critical value in a test at the 95% level with 240 d.f. = 205.1, "CHIINV(.95, 16*15)" **Results**: The test statistics of large and small markets are smaller than the critical value for rejecting H.					
Large vs. Small	10.8	109.7	76.5	72.4	10.8
1.4: Sample Bayesian information criterion for estimated models with alternative sets of covariates: other property types in the same city. **Results:** A first order model with no covariates minimizes the BIC for each property type.					
Covariate Set					
None	**7,353.4**	**7,828.4**	**6,583.1**	**5,844.1**	**7,346.4**
Apartment	-----	10,79.7	9,341.9	8,781.3	8,448.6
Hotel	8,343.3	-----	7,832.6	8,869.5	8,463.4
Industrial	9,841.8	10,792.7	-----	7,041.1	8,463.4
Office/	8,347.0	9,84.5	7,506.1	-----	8,439.0
Retail	9,816.9	8,669.4	9.922.3	6,908.4	-----

In taking an alternative route to establishing the order of the Markov chain, Berchtold (2006, page 51) recommends the estimating of alternative models, and then comparing of measures of model performance. Berchtold recommends the selecting of a model that gives the lowest Bayesian information criterion (BIC) value. It is a test statistic that decreases if added model parameters contribute sufficiently to justify added complexity, while the BIC increases otherwise. The BIC is determined by the log-likelihood of the estimated model, number of components in the likelihood function, and number of independent parameters needed. Table 1.2 shows the estimated BIC for alternative orders of Markov chains. For each property type, a first order model minimizes the sample BIC.

4.2 Empirical Tests for Ability to Pool Samples of Alternative Property Types

Once we select the Markov chain specification of each property type as being a first order model, it is natural to ask whether the processes are the same both qualitatively and quantitatively. If the cycle points of the property types come from the same Markov process, or processes that are very similar, then sample sizes can be doubled or tripled by pooling. Pooled data sets give more precision in estimated parameters because of reduced sampling error risk, but only if they do not become more random because they are not really from the same data generating process.

For this type of problem, Billingsley (1961, page 26) provides a chi-square test statistic for two samples:

$$\sum_{i,j} \frac{f_{i,.}g_{i,.}}{f_{i,j}+g_{i,j}}\left(\frac{f_{i,j}}{f_{i,.}} - \frac{g_{i,j}}{g_{i,.}}\right)^2,$$

where $f_{i,.}$ and $f_{i,j}$ are the same as defined above and apply to one sample, and $g_{i,.}$ and $g_{i,j}$ refer to the tally matrix of the second sample. Under the null hypothesis that both samples come from the same stochastic process, the test statistic has (16)(16-1) = 240 degrees of freedom in this case of sixteen possible states. We use a critical value of 205.1 in evaluating the chi-square sample test statistics reported in Table 1.3. With that critical value, the null hypothesis is rejected in each pair of property types tested.

Some more detail on the selection of the critical value is necessary because, if different critical values are appropriate, two pairs would lead us to different statistical decision-making. In testing this null hypothesis, there would be losses from Type I errors. That is, if we reject the null hypothesis when it is true that a pair of property types have exactly the same first order stochastic process, then we lose by failing to exploit the advantages of pooling samples. The loss seems larger if we make a Type II error in testing this null hypothesis. If we accept the null hypothesis, but the pair does not have the same Markov chain model, then losses would come from both believing that a pair of real estate property types moved together in that manner and pooling samples that should not be pooled. Given a sample, we can lower the probability of a Type II error by raising the selected probability of a Type I error. The critical value of 205.1 comes from setting the probability of a Type I error at .95, often called “alpha”. With that critical value, the null hypothesis is rejected in each pair of property types tested. If alpha is set at .90, then we could not reject the null hypothesis for the office-industrial pair of samples, while an alpha of .50 would add another pair, retail-apartments.

4.3 Empirical Tests for Pooling Within Property Types

Billingley’s test statistic may also allow us to test for homogeneity within a sample for a given property type. One such test that may be done from the market cycle data history in Mueller (1995) is for subsamples defined by the overall market size. For each property type, the largest markets that represent 50% of all square footage in the 54 markets studied are indicated with city names given with bold italic print font in the reports, while smaller markets are printed in normal fonts. Through the use of the same test statistic and critical value described above for the rest of Table 1.3, we fail to reject the null hypothesis that the large and small city sub samples have the same first order Markov chain process. When we decide that we may pool the subsamples, we have to add a caveat. With some property types having as few as ten large city markets, the subsample tally matrices do not meet the rule of thumb that the expected frequency of each cell should be at least five if all cells are equally likely.

4.4 Empirical Tests for Covariate Models

Table 1.4 shows the sample results for the Markov chain models for individual property types when another property type is paired with them as covariates in a Markov chain model. By using the cycle status of one city for a given property type as the variable to predict, the model uses the initial status of the same property type and the initial cycle status for a paired property type in the same city, and transition matrices for the two property types as covariates that are moving together. The covariate model may make better predictions, but is more complex and requires more parameters. If the pair of property types do move together at the same city level, then the BIC may be lower for the covariate model than for a simple Markov chain model.

As an example of interpreting Table 1.4, a simple, first order Markov chain model generates a sample BIC of 7,353.4 for Apartments—when there is no covariate. When the Hotel market cycle conditions of the cities are added to Apartments in a covariate model, the BIC does not decrease. The 8,343.3 BIC is higher because any improvement in prediction is overwhelmed by the added number of parameters in the covariate model.

None of the covariate models minimize the sample BICs relative to a simple, first order Markov chain model. This result would be the case if the property types have different Markov chain properties. It is consistent with the tests above that show that pooling samples from different property types is a risky modeling choice.

Thus, the Markov chain model specification calculations indicate that commercial real estate cycle points appear to fit a first order Markov chain model specification, with transition probabilities that do not change with respect to being in the large market subsample versus the small market subsample. The first order Markov chain properties differ across property types.

5. Applications

5.1 Application: Staying Time Distributions

An intuitive application available from the large library already developed in Markov chain theory will be valuable to analysts and shows how the processes remarkably differ across cycle points and the five property types. Directly from the estimates seen in the transition matrix in a first order Markov chain, we have parameters for a random variable--the count of consecutive quarters that a market in a Markov chain process will just stay in a current cycle point. The staying time is the count of quarters that the process may remain in cycle point s_i, here labelled as random variable q_i. In counting the initial period, this count is a strictly positive random variable. For a local market that is in cycle point i during an initial quarter, the probability of leaving s_i after having been there for only the initial period is one minus the probability of staying, $prob(q_i = 1) = (1 - p_{ii})$. Next, in order for the process to stay in s_i exactly two quarters, it would need to stay in quarter one, and then leave after the second. The probability of that sequence would be $(1 - p_{ii})\, p_{ii}$. The probability of staying in s_i exactly k quarters is $prob(q_i = k) = (1 - p_{ii})(p_{ii})^{k-1}$.

As an example of generating the distribution of staying time, note that 379 instances were observed for Apartment city markets that began in Cycle Point 1, and that 300 of these cases ended up with the city being in Cycle Point 1 in the next quarter, as shown in the tally matrix in Appendix Table 1 Panel A for Apartments. Thus, just below that tally matrix, the estimated transition matrix for Apartments shows $p_{11} = .7916$ as the quarter-to-quarter probability of staying in the trough of recession, Cycle Point 1. By using the formula, $prob(q_1 = 1) = (1 - p_{11}) = (1 - .7916) \approx .21$, as reported in Table 2 for Apartments initially in the trough of recession. For the other possible staying times, the exhibit reports calculations for $prob(q_1 = k) = (1 - p_{11})(p_{11})^{k-1}$. These probabilities are labeled p(q) in the exhibit, while the less-than-or-equal-to cumulative probabilities are labeled F(q). Many real estate analysts will find even more intuition for G(q), the more-than-or-equal-to cumulative probability.

Table 2 Staying Time Probabilities p(q); Less-Than-Or-Equal to Cumulative Probabilities, F(q); Greater-Than-Or-Equal to Cumulative Probabilities, G(q)

	Apartments			Hotel			Industrial			Office			Retail		
q	**p(q)**	**F(q)**	**G(q)**	**p(q)**	**F(q)**	**G(q)**	**p(q)**	**F(q)**	**G(q)**	**p(q)**	**F(q)**	**G(q)**	**p(q)**	**F(q)**	**G(q)**
Cycle Point 1: Trough of Recession															
1	.21	.21	1.00	.29	.29	1.00	.12	.12	1.00	.09	.09	1.00	.22	.22	1.00
2	.16	.37	.79	.21	.49	.71	.11	.23	.88	.08	.17	.91	.17	.39	.78
3	.13	.50	.63	.15	.64	.51	.09	.32	.77	.07	.24	.83	.13	.53	.61
4	.10	.61	.50	.10	.74	.36	.08	.41	.68	.07	.31	.76	.10	.63	.47
5	.08	.69	.39	.07	.82	.26	.07	.48	.59	.06	.37	.69	.08	.71	.37
6	.06	.75	.31	.05	.87	.18	.06	.54	.52	.06	.43	.63	.06	.78	.29
7	.05	.81	.25	.04	.91	.13	.06	.60	.46	.05	.48	.57	.05	.83	.22
8	.04	.85	.19	.03	.93	.09	.05	.65	.40	.05	.53	.52	.04	.86	.17
9	.03	.88	.15	.02	.95	.07	.04	.69	.35	.04	.57	.47	.03	.89	.14
10	.03	.90	.12	.01	.97	.05	.04	.73	.31	.04	.61	.43	.02	.92	.11
11	.02	.92	.10	.01	.98	.03	.03	.76	.27	.04	.64	.39	.02	.94	.08
12	.02	.94	.08	.01	.98	.02	.03	.79	.24	.03	.67	.36	.01	.95	.06
Cycle Point 8: Cost Effective New Construction															
1	.41	.41	1.00	.33	.33	1.00	.38	.38	1.00	.54	.54	1.00	.37	.37	1.00
2	.24	.65	.59	.22	.56	.67	.24	.62	.62	.25	.78	.46	.23	.61	.63
3	.14	.79	.35	.15	.70	.44	.15	.76	.38	.12	.90	.22	.15	.75	.39
4	.08	.88	.21	.10	.80	.30	.09	.85	.24	.05	.95	.10	.09	.84	.25
5	.05	.93	.12	.07	.87	.20	.06	.91	.15	.02	.98	.05	.06	.90	.16

(Continued…)

(Table 2 Continued)

q	Apartments p(q)	Apartments F(q)	Apartments G(q)	Hotel p(q)	Hotel F(q)	Hotel G(q)	Industrial p(q)	Industrial F(q)	Industrial G(q)	Office p(q)	Office F(q)	Office G(q)	Retail p(q)	Retail F(q)	Retail G(q)
Cycle Point 8: Cost Effective New Construction															
6	.03	.96	.07	.04	.91	.13	.03	.94	.09	.01	.99	.02	.04	.94	.10
7	.02	.97	.04	.03	.94	.09	.02	.96	.06	.01	1.00	.01	.02	.96	.06
8	.01	.98	.03	.02	.96	.06	.01	.98	.04	.00	1.00	.00	.01	.98	.04
9	.01	.99	.02	.01	.97	.04	.01	.99	.02	.00	1.00	.00	.01	.98	.02
10	.00	.99	.01	.01	.98	.03	.01	.99	.01	.00	1.00	.00	.01	.99	.02
11	.00	1.00	.01	.01	.99	.02	.00	.99	.01	.00	1.00	.00	.00	.99	.01
12	.00	1.00	.00	.00	.99	.01	.00	1.00	.01	.00	1.00	.00	.00	1.00	.01
Cycle Point 11: Equilibrium Growth in Supply and Demand															
1	.38	.38	1.00	.45	.45	1.00	.31	.31	1.00	.35	.35	1.00	.30	.30	1.00
2	.24	.62	.62	.25	.70	.55	.22	.53	.69	.23	.58	.65	.21	.51	.70
3	.15	.76	.38	.14	.84	.30	.15	.68	.47	.15	.73	.42	.15	.66	.49
4	.09	.85	.24	.07	.91	.16	.10	.78	.32	.10	.82	.27	.10	.76	.34
5	.06	.91	.15	.04	.95	.09	.07	.85	.22	.06	.89	.18	.07	.83	.24
6	.03	.94	.09	.02	.97	.05	.05	.90	.15	.04	.93	.11	.05	.88	.17
7	.02	.97	.06	.01	.99	.03	.03	.93	.10	.03	.95	.07	.04	.92	.12
8	.01	.98	.03	.01	.99	.01	.02	.95	.07	.02	.97	.05	.02	.94	.08
9	.01	.99	.02	.00	1.00	.01	.02	.97	.05	.01	.98	.03	.02	.96	.06
10	.01	.99	.01	.00	1.00	.00	.01	.98	.03	.01	.99	.02	.01	.97	.04
11	.00	.99	.01	.00	1.00	.00	.01	.98	.02	.00	.99	.01	.01	.98	.03
12	.00	1.00	.01	.00	1.00	.00	.00	.99	.02	.00	.99	.01	.01	.99	.02

Note: q is the Number of Quarters of Consecutive Location at the Cycle Point, Given That a Market Is Now in a Specified Cycle Point

For example, an investor who bought into an Apartment market in the trough of a recession could anticipate .10 as the probability of staying exactly four quarters and a .61 probability that the market would stay in those conditions for four or fewer quarters. It would seem more ominous to interpret the scenario as presenting a .50 probability of staying in the point four or more quarters.

Across property types, there is notable variation in the staying time probabilities. In the trough of recession, Apartments and Retail have comparable probabilities, while Industrial and Office are close to half of those, and Hotel is much higher at q = 1, at the one quarter level. Hotels have consistently higher less-than-or-equal-to cumulative staying probabilities for investments made in the trough of recession, while Industrial and Office are much lower than Apartments and Retail, which fall in between.

With the use of the greater-than-or-equal-to staying time probabilities at the fourth quarter point, Apartments and Retail are very close, .50 and .47, but Industrial and Office prospects for a year or more in the trough are .68 and .76 respectively. At the twelfth quarter, Apartments, Hotels and Retail have negligible greater-than-or-equal-to probabilities, but Industrial and Office still have .24 and .36 probabilities of even longer stays at Cycle Point 1, which is the trough.

The probabilities vary remarkably across initial cycle conditions. If the Apartment investment was made in Cycle Point 8--the first point of cost feasible new construction in the expansion stage of the cycle, then $p_{88} = .5923$, and $prob(q_8 = 1) = (1-p_{88}) \approx .41$, as reported in the middle of the column for Apartments in Table 2. That is just short of double the analogous probability calculated for an investment made in the recession trough. Table 2 also details the calculations for investments made at Cycle Point 11, where demand and supply are growing in equilibrium. Staying time probabilities there are also notably higher than trough of recession values.

Investors would prefer a much different pattern. High probabilities of staying in bad real estate cycle points for only one quarter are attractive. This is because it means that movement away is more likely and longer stays are less likely. On the other hand, high probabilities of staying in the more favorable cycle points for only one quarter would be unattractive--long, profitable stays are less likely.

5.2 Application: Mean Staying Time

A related application has an even more intuitive application, but is still only based on the p_{ii} parameters given by the transition matrices. When a random variable has the probability, $prob(q_i = k) = (1 - p_{ii})\,(p_{ii})^{k-1}$, it is said to have geometric distribution. Although the shape of the distribution differs for each property type and at each cycle point, it is qualitatively the same, depending on only one parameter of the distribution, the estimate of p_{ii}. For all geometric distributions, expected value and variances for these random variables, q_i, are

$$E[q_i] = \frac{1}{1-p_{ij}} \quad \text{and} \quad Var[q_i] = \frac{p_{ij}}{(1-p_{ij})^2}$$

The expected value is "mean staying time". The estimates for the mean staying times and the standard deviations for all sixteen real estate cycle points and each property type are provided in Table 3.

Table 3 Mean Staying Times (and Standard Deviations) in Quarters

Cycle Point	Apartment	Hotel	Industrial	Office	Retail
1	4.8 (4.3)	3.5 (2.9)	8.1 (7.6)	11.2 (1.7)	4.5 (4.0)
2	4.0 (3.5)	3.2 (2.7)	3.9 (3.3)	4.3 (3.8)	3.6 (3.1)
3	3.5 (2.9)	2.9 (2.4)	4.1 (3.5)	3.4 (2.9)	3.6 (3.0)
4	3.3 (2.8)	2.6 (2.0)	3.6 (3.1)	2.7 (2.1)	2.7 (2.2)
5	3.7 (3.2)	2.4 (1.8)	2.6 (2.0)	2.4 (1.8)	2.4 (1.9)
6	4.2 (3.6)	2.6 (2.1)	3.0 (2.4)	2.5 (2.0)	3.9 (3.4)
7	3.0 (2.4)	2.9 (2.4)	2.5 (1.9)	2.5 (1.9)	4.1 (3.6)
8	2.5 (1.9)	3.0 (2.4)	2.6 (2.1)	1.9 (1.3)	2.7 (2.1)
9	3.0 (2.5)	2.8 (2.2)	3.3 (2.8)	2.9 (2.3)	3.1 (2.5)
10	3.3 (2.7)	2.0 (1.5)	2.7 (2.2)	2.5 (1.9)	3.3 (2.8)
11	2.6 (2.1)	2.2 (1.6)	3.2 (2.6)	2.8 (2.3)	2.5 (2.0)
12	2.6 (2.1)	2.4 (1.9)	1.8 (1.2)	2.0 (1.5)	2.5 (1.9)
13	2.0 (1.4)	2.1 (1.5)	1.6 (1.0)	1.6 (1.0)	2.2 (1.7)
14	1.8 (1.2)	2.6 (2.0)	1.7 (1.1)	1.7 (1.0)	2.3 (1.7)
15	2.1 (1.5)	2.4 (1.9)	1.9 (1.3)	2.1 (1.5)	2.6 (2.0)
16	2.1 (1.6)	2.0 (1.4)	1.4 (.8)	2.0 (1.4)	1.9 (1.3)

Again, the real estate investor has to understand the definition of "good" changes. A long mean staying time is "not good" if the cycle point is a bad one, while a long mean staying time is "good" in favorable cycle points. The systematic pattern of Cycle Point 1 of higher mean staying times means that investors endure the trough of recession for longer periods than seen for other cycle points. Each of the five property types have their longest mean staying time at the troughs of the recessions. Moreover, industrial and office markets have much longer mean staying times in very poor trough conditions. These property types are less attractive in those cycle points than other property types with mean staying times that are half or one third of those of office and industrial. On the other hand, the mean staying times of office and industrial are the most attractive among the set of five for the most profitable cycle point that represents the highest occupancies and rent conditions, Cycle Point 11. Most of the shortest mean staying times are in hyper supply and recession phases, with the range across property types being narrow in these cycle points.

The dramatic change of mean staying times across cycle points also confirms the perception of many real estate analysts about the cycle. The change in cycle conditions sometimes seem to move very slowly or even pause, and then change rapidly as the cycle moves to other stages.

6. Concluding Comments

While this paper deals with the attractive applications of Markov chain analysis in commercial real estate cycle analysis, it has limitations. Markov models are probability models that have no economic or real estate fundamental inputs beyond having justifiable transition probabilities. In the application here, those probabilities are empirical estimates, but real estate analysts with sound judgment may use subjective probabilities for the transition coefficients. While the Markov models may generate useful forecasts, which are enough to justify scientific application, they do not explain what has happened or provide economic arguments for what will happen.

It will always be essential to understand the fundamental local real estate market drivers of supply and demand (Wheaton and Torto, 1988; Mueller, 1999; Holt and Mills, 2000; Mueller and Laposa, 1994 and 1995) , the powers of world financial variables (Mueller, 1995), and macroeconomic environmental factors (Pyhrrr et al., 1990 and 1990a; Pyhrr et al., 1996). A recent review of the fundamentals of real estate cycles appears in Evans and Mueller (2013). It seems impossible that econometric models of these variables will ever be supplanted by Markov chain models for short term forecasts in terms of accuracy and believability by knowledgeable users of the forecasts, and for the purposes of explanation of past trends and fluctuations.

Econometric models are expensive in terms of the quality of the analyst required and the length, breadth and precision of the data sets needed for valid forecasting. Markov chain models are “simple” in the sense that they need information about current market cycle conditions and historic transition probabilities. These transition probabilities can be based on historic patterns over cycles or simple subjective judgment. The transition probabilities reported here are empirical estimates based on observed historic cycle trends. The first order Markov chain forecasting calculations are new to most real estate analysts, but not as hard to learn as econometric methods.

The specification of the models as first order Markov chains, surprisingly, is not the same as rejection of the common perception of real estate cycles having momentum, pauses and patterns that seem to be multi-period phenomena instead of the short memory stochastic process of a first order model. First, the model generates notable changes from cycle point to cycle point with respect to mean first passage time, thus modeling the behavior that cycle analysts see as stagnation and momentum. In addition, the Mueller model of cycle points is predicated on these qualitative factors. That is, high occupancy that is growing defines a different cycle point than the same high occupancy that is declining or growing more slowly. Rent levels that are growing faster or slower also distinguish different cycle points. The evidence that led to the acceptance of first order Markov chain specifications could be interpreted as asserting that the definitions of the cycle points are essentially correct.

Each of the five property types have their longest mean staying time at the troughs of recessions. Moreover, industrial and office markets have much longer mean staying times in very poor trough conditions. These property types are less attractive in those cycle points than other property types that have mean staying times that are half or one third of those of office and industrial. On the other hand, the mean staying times of office and industrial are the most attractive among the set of five for the most profitable cycle point that represents the highest occupancies and rent conditions, Cycle Point 11. Most of the shortest mean staying times are in hyper supply and recession phases, with the range across property types being narrow in these cycle points. Analysts and investors should be able to use this research to better estimate future occupancy and rent estimates in their DCF models.

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Appendix Table 1 Cycle Conditions of Apartment Markets

Panel A: Tallies for One-Quarter Transitions across Cycle Points 1—16 in Apartments

$f_{i,j}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$f_{i,\cdot}$
1	300	72	5	1	0	0	0	0	0	0	0	0	0	0	1	0	379
2	9	291	63	10	5	6	0	0	0	0	0	0	0	0	1	3	388
3	0	5	163	32	12	10	3	0	0	0	0	0	0	0	4	0	229
4	0	0	5	102	23	11	2	0	0	0	0	0	0	0	3	0	146
5	0	0	0	2	104	22	5	2	0	0	0	0	0	1	6	0	142
6	0	0	0	0	2	142	27	4	1	2	1	0	0	8	0	0	187
7	0	0	0	1	0	1	88	32	3	2	0	2	0	3	0	0	132
8	0	0	0	1	0	0	6	77	32	3	2	2	6	0	1	0	130
9	0	0	0	0	0	0	3	7	139	35	12	9	2	1	0	0	208
10	0	0	0	0	0	0	0	2	12	181	40	25	0	1	0	0	261
11	0	0	0	0	0	1	0	1	0	10	107	47	6	1	0	0	173
12	0	0	0	0	0	0	2	0	3	19	8	170	62	9	2	0	275
13	0	0	1	0	0	0	1	0	2	0	0	12	80	55	8	3	162
14	5	1	2	2	1	2	0	0	0	0	0	2	3	67	58	6	149
15	19	6	1	0	0	1	0	0	0	0	0	0	2	3	91	53	176
16	47	16	0	0	1	0	0	0	0	0	0	0	0	0	1	74	139
$f_{\cdot,j}$	380	391	240	151	148	196	137	125	192	252	170	269	161	149	176	139	3,276

(Continued…)

(Appendix Table 1 Continued)

Panel B: Relative Frequencies for One-Quarter Transitions across Cycle Points 1—16 in Apartments

$p_{i,j}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.7916	.1900	.0132	.0026	0	0	0	0	0	0	0	0	0	0	.0026	0
2	.0232	.7500	.1624	.0258	.0129	.0155	0	0	0	0	0	0	0	0	.0026	.0077
3	0	.0218	.7118	.1397	.0524	.0437	.0131	0	0	0	0	0	0	0	.0175	0
4	0	0	.0342	.6986	.1575	.0753	.0137	0	0	0	0	0	0	0	.0205	0
5	0	0	0	.0141	.7324	.1549	.0352	.0141	0	0	0	0	0	.0070	.0423	0
6	0	0	0	0	.0107	.7594	.1444	.0214	.0053	.0107	.0053	0	0	.0428	0	0
7	0	0	0	.0076	0	.0076	.6667	.2424	.0227	.0152	0	.0152	0	.0227	0	0
8	0	0	0	.0077	0	0	.0462	.5923	.2462	.0231	.0154	.0154	.0462	.0000	.0077	0
9	0	0	0	0	0	0	.0144	.0337	.6683	.1683	.0577	.0433	.0096	.0048	0	0
10	0	0	0	0	0	0	0	.0077	.0460	.6935	.1533	.0958	0	.0038	0	0
11	0	0	0	0	0	.0058	0	.0058	0	.0578	.6185	.2717	.0347	.0058	0	0
12	0	0	0	0	0	0	.0073	0	.0109	.0691	.0291	.6182	.2255	.0327	.0073	0
13	0	0	.0062	0	0	0	.0062	0	.0123	0	0	.0741	.4938	.3395	.0494	.0185
14	.0336	.0067	.0134	.0134	.0067	.0134	0	0	0	0	0	.0134	.0201	.4497	.3893	.0403
15	.1080	.0341	.0057	0	0	.0057	0	0	0	0	0	0	.0114	.0170	.5170	.3011
16	.3381	.1151	0	0	.0072	0	0	0	0	0	0	0	0	0	.0072	.5324

Appendix Table 2 Cycle Condition of Hotel Markets

Panel A: Tallies for One-Quarter Transitions across Cycle Points 1—16 in Hotels

$f_{i,j}$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	$f_{i,\cdot}$
1	229	91	2	0	0	0	0	0	0	0	0	0	0	0	0	0	322
2	27	281	95	2	1	0	0	0	0	0	0	0	0	0	0	1	407
3	0	12	185	76	6	0	0	0	0	0	0	0	0	0	0	2	281
4	0	4	3	132	56	16	0	1	0	0	0	0	0	0	4	0	216
5	1	0	0	9	110	50	8	0	0	0	0	0	0	0	11	0	189
6	1	0	0	4	18	168	52	7	0	0	0	0	0	18	1	2	271
7	0	0	0	0	1	22	136	34	2	0	1	0	3	8	0	0	207
8	0	0	0	0	1	5	12	134	31	2	3	2	4	7	0	0	201
9	0	0	0	0	0	1	0	11	95	22	7	6	6	0	0	0	148
10	0	0	0	0	0	0	0	0	8	50	29	11	0	0	0	0	98
11	0	0	0	0	0	0	0	1	2	7	80	49	3	4	0	0	146
12	1	1	0	1	0	0	0	0	1	5	20	128	48	11	0	2	218
13	0	1	0	0	0	2	2	3	3	0	1	16	82	31	6	12	159
14	8	16	0	0	0	12	0	0	1	0	0	0	12	130	28	6	213
15	22	3	0	1	3	3	0	0	0	0	0	0	0	3	72	15	122
16	33	5	1	0	0	0	0	0	0	0	0	0	0	1	0	38	78
$f_{\cdot,j}$	322	414	286	225	196	279	210	191	143	86	141	212	158	213	122	78	3,276

(Continued…)

(Appendix Table 2 Continued)

Panel B: Relative Frequencies for One-Quarter Transitions across Cycle Points 1—16 in Hotels

$p_{i,j}$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	.7112	.2826	.0062	0	0	0	0	0	0	0	0	0	0	0	0	0
2	.0663	.6904	.2334	.0049	.0025	0	0	0	0	0	0	0	0	0	0	.0025
3	0	.0427	.6584	.2705	.0214	0	0	0	0	0	0	0	0	0	0	.0071
4	0	.0185	.0139	.6111	.2593	.0741	0	.0046	0	0	0	0	0	0	.0185	.0000
5	.0053	0	0	.0476	.5820	.2646	.0423	0	0	0	0	0	0	0	.0582	.0000
6	.0037	0	0	.0148	.0664	.6199	.1919	.0258	0	0	0	0	0	.0664	.0037	.0074
7	0	0	0	0	.0048	.1063	.6570	.1643	.0097	0	.0048	0	.0145	.0386	0	.0000
8	0	0	0	0	.0050	.0249	.0597	.6667	.1542	.0100	.0149	.0100	.0199	.0348	0	.0000
9	0	0	0	0	0	.0068	0	.0743	.6419	.1486	.0473	.0405	.0405	0	0	.0000
10	0	0	0	0	0	0	0	0	.0816	.5102	.2959	.1122	0	0	0	.0000
11	0	0	0	0	0	0	0	.0068	.0137	.0479	.5479	.3356	.0205	.0274	0	.0000
12	.0046	.0046	0	.0046	0	0	0	0	.0046	.0229	.0917	.5872	.2202	.0505	0	.0092
13	0	.0063	0	0	0	.0126	.0126	.0189	.0189	0	.0063	.1006	.5157	.1950	.0377	.0755
14	.0376	.0751	0	0	0	.0563	0	0	.0047	0	0	0	.0563	.6103	.1315	.0282
15	.1803	.0246	0	.0082	.0246	.0246	0	0	0	0	0	0	0	.0246	.5902	.1230
16	.4231	.0641	.0128	0	0	0	0	0	0	0	0	0	0	.0128	0	.4872

Appendix Table 3 Cycle Conditions of Industrial Markets

Panel A: Tallies for One-Quarter Transitions across Cycle Points 1—16 in Industrial

$f_{i,j}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$f_{i,\cdot}$
1	739	96	10	0	1	0	0	0	0	0	0	0	0	0	0	1	847
2	21	324	67	8	2	2	0	0	0	0	0	0	0	0	0	12	436
3	0	9	237	55	2	4	0	0	0	0	0	0	0	0	9	0	316
4	0	0	6	145	19	15	0	0	0	0	0	0	0	0	15	0	200
5	0	0	0	4	38	10	5	0	0	0	0	0	0	0	5	0	62
6	0	0	0	1	5	69	20	4	1	0	0	0	0	0	4	0	104
7	0	0	0	0	0	3	45	16	2	2	0	0	6	2	0	0	76
8	0	0	0	0	0	0	4	49	18	5	2	2	0	0	0	0	80
9	0	0	0	0	0	0	4	8	150	24	6	16	2	0	0	0	210
10	0	0	0	0	0	0	0	0	15	121	36	11	2	0	0	0	185
11	0	0	0	0	0	0	0	0	5	11	176	60	4	1	0	0	257
12	0	0	0	0	0	0	0	0	1	10	24	85	52	11	7	0	190
13	1	1	2	0	0	0	0	0	2	4	6	11	42	20	19	2	110
14	2	5	2	0	0	0	0	0	0	0	0	1	1	23	17	7	58
15	7	5	1	0	0	0	0	0	0	0	1	0	0	1	72	63	150
16	76	7	0	0	0	0	0	0	0	0	0	0	0	0	2	36	121
$f_{\cdot,j}$	846	447	325	213	67	103	78	77	194	177	251	186	109	58	150	121	3,402

(Continued…)

(Appendix Table 3 Continued)

Panel B: Relative Frequencies for One-Quarter Transitions across Cycle Points 1—16 in Industrial

$p_{i,j}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.8725	.1133	.0118	0	.0012	0	0	0	0	0	0	0	0	0	0	.0012
2	.0482	.7431	.1537	.0183	.0046	.0046	0	0	0	0	0	0	0	0	0	.0275
3	0	.0285	.7500	.1741	.0063	.0127	0	0	0	0	0	0	0	0	.0285	0
4	0	0	.0300	.7250	.0950	.0750	0	0	0	0	0	0	0	0	.0750	0
5	0	0	0	.0645	.6129	.1613	.0806	0	0	0	0	0	0	0	.0806	0
6	0	0	0	.0096	.0481	.6635	.1923	.0385	.0096	0	0	0	0	0	.0385	0
7	0	0	0	0	0	.0395	.5921	.2105	.0263	.0263	0	0	.0789	.0263	0	0
8	0	0	0	0	0	0	.0500	.6125	.2250	.0625	.0250	.0250	0	0	0	0
9	0	0	0	0	0	0	.0190	.0381	.7143	.1143	.0286	.0762	.0095	0	0	0
10	0	0	0	0	0	0	0	0	.0811	.6541	.1946	.0595	.0108	0	0	0
11	0	0	0	0	0	0	0	0	.0195	.0428	.6848	.2335	.0156	.0039	0	0
12	0	0	0	0	0	0	0	0	.0053	.0526	.1263	.4474	.2737	.0579	.0368	0
13	.0091	.0091	.0182	0	0	0	0	0	.0182	.0364	.0545	.1000	.3818	.1818	.1727	.0182
14	.0345	.0862	.0345	0	0	0	0	0	0	0	0	.0172	.0172	.3966	.2931	.1207
15	.0467	.0333	.0067	0	0	0	0	0	0	0	.0067	0	0	.0067	.4800	.4200
16	.6281	.0579	0	0	0	0	0	0	0	0	0	0	0	0	.0165	.2975

Appendix Table 4 Cycle Conditions of Office Markets

Panel A: Tallies for One-Quarter Transitions across Cycle Points 1—16 in Offices

$f_{i,j}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$f_{i,\cdot}$
1	1,048	87	2	1	0	0	0	0	0	0	0	0	0	0	0	6	1,144
2	26	355	59	3	0	0	0	0	0	0	0	0	0	0	0	12	455
3	0	14	153	26	10	1	0	0	0	0	0	0	0	0	11	0	215
4	0	1	12	70	21	2	0	0	0	0	0	0	0	0	5	0	111
5	0	0	1	8	66	28	1	0	0	0	0	0	0	0	10	0	114
6	0	0	0	0	9	58	18	6	0	0	0	0	0	4	1	0	96
7	0	0	0	2	3	4	49	21	4	0	0	0	0	0	0	0	83
8	0	0	0	0	0	1	6	35	29	1	0	0	1	1	0	0	74
9	0	0	0	0	0	0	0	4	98	32	11	6	1	0	0	0	152
10	0	0	0	0	0	0	0	0	6	70	35	9	0	0	0	0	120
11	0	0	0	0	0	0	0	0	1	7	152	58	7	1	0	0	226
12	0	0	0	0	0	0	0	0	2	3	23	96	46	6	2	0	178
13	0	0	0	0	0	0	0	1	0	0	3	9	40	38	11	1	103
14	1	0	0	0	0	1	0	0	0	0	0	0	7	32	26	17	84
15	14	4	0	0	0	0	0	0	0	0	0	0	0	1	75	47	141
16	74	9	0	0	0	0	0	0	0	0	0	0	0	0	0	86	169
$f_{\cdot,j}$	1,163	470	227	110	109	95	74	67	140	113	224	178	102	83	141	169	3,465

(Continued…)

(Appendix Table 4 Continued)

Panel B: Relative Frequencies for One-Quarter Transitions across Cycle Points 1—16 in Offices

$p_{i,j}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.9161	.0760	.0017	.0009	0	0	0	0	0	0	0	0	0	0	0	.0052
2	.0571	.7802	.1297	.0066	0	0	0	0	0	0	0	0	0	0	0	.0264
3	0	.0651	.7116	.1209	.0465	.0047	0	0	0	0	0	0	0	0	.0512	0
4	0	.0090	.1081	.6306	.1892	.0180	0	0	0	0	0	0	0	0	.0450	0
5	0	0	.0088	.0702	.5789	.2456	.0088	0	0	0	0	0	0	0	.0877	0
6	0	0	0	0	.0938	.6042	.1875	.0625	0	0	0	0	0	.0417	.0104	0
7	0	0	0	.0241	.0361	.0482	.5904	.2530	.0482	0	0	0	0	0	0	0
8	0	0	0	0	0	.0135	.0811	.4730	.3919	.0135	0	0	.0135	.0135	0	0
9	0	0	0	0	0	0	0	.0263	.6447	.2105	.0724	.0395	.0066	0	0	0
10	0	0	0	0	0	0	0	0	.0500	.5833	.2917	.075	0	0	0	0
11	0	0	0	0	0	0	0	0	.0044	.0310	.6726	.2566	.0310	.0044	0	0
12	0	0	0	0	0	0	0	0	.0112	.0169	.1292	.5393	.2584	.0337	.0110	0
13	0	0	0	0	0	0	0	.0097	0	0	.0291	.0874	.3883	.3689	.1068	.0097
14	.0119	0	0	0	0	.0119	0	0	0	0	0	0	.0833	.3810	.3095	.2024
15	.0993	.0284	0	0	0	0	0	0	0	0	0	0	0	.0071	.5319	.3333
16	.4379	.0533	0	0	0	0	0	0	0	0	0	0	0	0	0	.5089

Appendix Table 5 Cycle Conditions of Retail Markets

Panel A: Tallies for One-Quarter Transitions across Cycle Points 1—16 in Retail

$f_{i,j}$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	$f_{i,\cdot}$
1	258	57	15	0	0	0	0	0	0	0	0	1	0	0	0	0	331
2	9	153	32	11	1	0	0	0	0	0	0	0	0	0	1	4	211
3	1	3	147	37	4	0	0	0	0	0	0	0	0	0	12	0	204
4	0	1	17	127	38	14	0	0	0	0	0	0	0	0	3	0	200
5	0	1	0	22	105	39	6	0	0	0	0	1	0	4	1	0	179
6	0	0	0	1	28	186	32	2	0	0	0	0	0	0	1	0	250
7	0	0	0	0	3	3	131	27	6	0	0	1	0	2	0	0	173
8	0	0	0	0	0	1	5	64	22	2	4	1	2	1	0	0	102
9	0	0	0	0	0	0	0	5	99	23	10	10	0	0	0	0	147
10	0	0	0	0	0	0	1	0	10	156	48	7	0	1	0	0	223
11	0	0	0	0	0	0	0	0	0	20	155	78	1	1	0	0	255
12	0	0	0	0	0	0	0	1	3	8	31	196	73	15	1	0	328
13	0	0	0	2	4	4	0	0	1	2	1	20	109	45	8	1	197
14	2	0	0	2	1	7	0	1	0	0	0	4	7	95	46	6	171
15	20	1	3	1	1	2	0	0	0	0	0	0	1	5	123	43	200
16	45	9	1	0	0	0	0	0	0	0	0	0	0	0	0	50	105
$f_{\cdot,j}$	335	225	215	203	185	256	175	100	141	211	249	319	193	169	196	104	3,276

(Continued…)

(Appendix Table 5 Continued)

Panel B: Relative Frequencies for One-Quarter Transitions across Cycle Points 1—16 in Retail

$p_{i,j}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	.7795	.1722	.0453	0	0	0	0	0	0	0	0	.0030	0	0	0	0
2	.0427	.7251	.1517	.0521	.0047	0	0	0	0	0	0	0	0	0	.0047	.0190
3	.0049	.0147	.7206	.1814	.0196	0	0	0	0	0	0	0	0	0	.0588	0
4	0	.0050	.0850	.6350	.1900	.0700	0	0	0	0	0	0	0	0	.0150	0
5	0	.0056	0	.1229	.5866	.2179	.0335	0	0	0	0	0	0	.0223	.0056	0
6	0	0	0	.0040	.1120	.7440	.1280	.0080	0	0	0	.0056	0	0	.0040	0
7	0	0	0	0	.0173	.0173	.7572	.1561	.0347	0	0	.0058	0	.0116	0	0
8	0	0	0	0	0	.0098	.0490	.6275	.2157	.0196	.0392	.0098	.0196	.0098	0	0
9	0	0	0	0	0	0	0	.0340	.6735	.1565	.0680	.0680	0	0	0	0
10	0	0	0	0	0	0	.0045	0	.0448	.6996	.2152	.0314	0	.0045	0	0
11	0	0	0	0	0	0	0	0	0	.0784	.6078	.3059	.0039	.0039	0	0
12	0	0	0	0	0	0	0	.0030	.0091	.0244	.0945	.5976	.2226	.0457	.0030	0
13	0	0	0	.0102	.0203	.0203	0	0	.0051	.0102	.0051	.1015	.5533	.2284	.0406	.0051
14	.0117	0	0	.0117	.0058	.0409	0	.0058	0	0	0	.0234	.0409	.5556	.2690	.0351
15	.1000	.0050	.0150	.0050	.0050	.0100	0	0	0	0	0	0	.0050	.0250	.6150	.2150
16	.4286	.0857	.0095	0	0	0	0	0	0	0	0	0	0	0	0	.4762